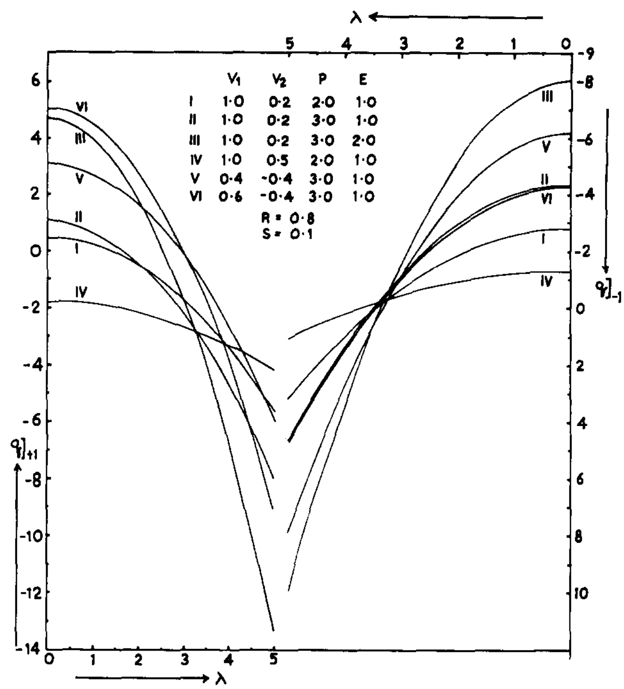

FIG. 7. Rate of heat transfer at the discs.

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EVALUATION OF THE GEOMETRIC MEAN TRANSMITTANCE AND TOTAL ABSORPTANCE FOR TWO-DIMENSIONAL SYSTEMS

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NOMENCLATURE

a_λ,	absorption coefficient;
A_1, A_2,	area elements;
D,	length defined in Fig. 3;
E_n,	exponential function;
$F(x, 1/2^{1/2})$,	elliptic function of the first kind;
F_{i-j},	shape factor between areas A_i and A_j;
H,	$(R^2 + z^2)^{1/2}$;
$\mathbf{n}_1$, $\mathbf{n}_2$,	unit vectors defined in Fig. 1;
$\mathbf{r}_1$,	unit vector defined in Fig. 1;
r,	distance between dA_1 and dA_2;
R,	length defined in Fig. 2;
R_{i-j},	weak-band geometric mean beam length;
S_{i-j},	strong-band geometric mean beam length;
W_{i-j},	very-strong-band geometric mean beam length;
θ,	angle defined in Fig. 2;
ϕ,	angle defined in Fig. 3;
$\tau_{\lambda,d1-2}$,	geometric-mean transmittance.

INTRODUCTION

In a recent work [1], the evaluation of the geometric mean transmittance and total absorptance between an infinitesimal area and an arbitrary finite area with an intervening absorbing and emitting medium was shown to be greatly simplified by a mathematical identity which has never been recognized before. Solutions for three specific geometries were generated and identified as 'fundamental solutions'. Solutions for many 3-dim. systems were shown to be readily generated from these fundamental solutions by superposition. While these fundamental solutions are clearly of great value for 3-dim. application, they are inconvenient and difficult to use for systems with 2-dim. geometry. The objective of this work is to develop the corresponding 2-dim. fundamental solutions for this important practical problem in radiative heat transfer.

MATHEMATICAL FORMULATION

In reference [1], the geometric mean transmittance and the geometric mean beam length in the three optical limits (weak, strong and very strong) between an infinitesimal area dA_1 and A_2 with orientation as shown in Fig. 1 were shown to reduce to

$$\pi F_{d1-2}\tau_{\lambda,d1-2} = -\int_{S_2}\frac{E_3(a_\lambda r)}{r}(\mathbf{r}_1\times\mathbf{n}_1)\cdot d\mathbf{S}_2 + \int_{A_2}(\mathbf{n}_1\cdot\mathbf{n}_2)\frac{a_\lambda}{r}E_2(a_\lambda r)\,dA_2 \quad (1)$$

$$\pi F_{d1-2}R_{d1-2} = -\int_{S_2}(\mathbf{r}_1\times\mathbf{n}_1)\cdot d\mathbf{S}_2 - \int_{A_2}(\mathbf{n}_1\cdot\mathbf{n}_2)\frac{dA_2}{r} \quad (2)$$

$$\pi F_{d1-2}S_{d1-2}^{1/2} = -2/3\int_{S_2}\frac{(\mathbf{r}_1\times\mathbf{n}_1)\cdot d\mathbf{S}_2}{r^{1/2}} - 1/3\int_{A_2}\frac{(\mathbf{n}_1\cdot\mathbf{n}_2)\,dA_2}{r^{3/2}} \quad (3)$$

$$\pi F_{d1-2}\ln W_{d1-2} = -1/2\int_{S_2}\left(\frac{\ln r}{r}+\frac{1}{2r}\right)(\mathbf{r}_1\times\mathbf{n}_1)\cdot d\mathbf{S}_2 - 1/2\int_{A_2}(\mathbf{n}_1\cdot\mathbf{n}_2)\frac{dA_2}{r^2} \quad (4)$$

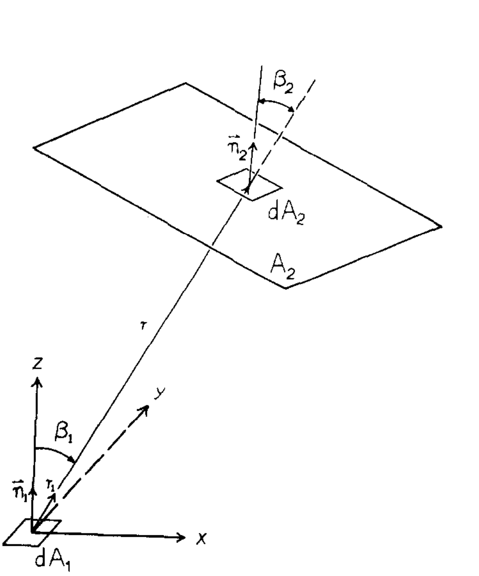

FIG. 1. Coordinate system used in equations (1)–(4).

In the above expression, $\tau_{\lambda,d1-2}$, R_{d1-2}, S_{d1-2}, and W_{d1-2} stand for the geometric mean transmittance, the geometric mean beam length in the weak-band, strong-band, and very-strong-band limits between dA_1 and dA_2, respectively. F_{d1-2} is the shape factor between dA_1 and A_2; $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors normal to areas dA_1 and dA_2; $\mathbf{r}_1$ is a unit vector pointing away from dA_1 to dA_2; r is the distance between the two infinitesimal areas; a_λ is the absorption coefficient and $E_3(a_\lambda r)$ is the exponential integral function given by

$$E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n}\,dt\,. \quad (5)$$

For 2-dim. application, the relevant geometry is one in which both dA_1 and A_2 are infinitely extended in one direction. In fact, the 2-dim. fundamental solutions are generated by evaluating equations (1)–(4) for the geometry as shown in Fig. 2 where dA_1 is an infinitesimal area at the origin and A_2 is an infinitely long circular arc of radius R. (Note that solutions with dA_1 being an infinitely long strip are identical to that of Fig. 2 because of symmetry. The choice of A_2 being an infinitely long circular arc instead of an infinite strip of finite width as in 2-dim. shape factor calculation [2] is for the convenience of obtaining closed-form solution.) It can be readily shown that solutions with A_2 being an arbitrary 2-dim. object can be approximated as sums and differences of these fundamental solutions by superposition.

Utilizing the coordinate system as shown in Fig. 2, the vectors $\mathbf{n}_1$, $\mathbf{n}_2$, and $\mathbf{r}_1$ can be written as

$$\mathbf{n}_1 = \mathbf{j} \quad (6)$$

$$\mathbf{n}_2 = \cos\theta\mathbf{i} + \sin\theta\mathbf{j} \quad (7)$$

$$\mathbf{r}_1 = (R\cos\theta\mathbf{i} + R\sin\theta\mathbf{j} + z\mathbf{k})/\mathbf{H} \quad (8)$$

with $H = (R^2+z^2)^{1/2}$ and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ being the unit vector in the x-, y-, and z-directions, respectively. Substituting equations (6)–(8) into equation (1), the geometric mean transmittance becomes

$$\pi F_{d1-2}\tau_{\lambda,d1-2} = 2R(\cos\theta_1-\cos\theta_2)\times\left[\int_0^\infty\frac{E_3(a_\lambda H)}{H^2}dz + a_\lambda\int_0^\infty\frac{E_2(H)}{H}dz\right] \quad (9)$$

Using the result of shape factor calculation [2]

$$F_{d1-2} = 1/2(\cos\theta_1-\cos\theta_2) \quad (10)$$

and the approximation

$$E_3(z) = 1/2\,e^{-3/2z} \quad (11)$$

$$E_2(z) = 3/4\,e^{-3/2z} \quad (12)$$

equation (9) can be integrated in closed-form to yield

$$\tau_{\lambda d1-2} = 1 + \frac{2}{\pi}\beta K_0(\beta) - \beta[K_0(\beta)L_{-1}(\beta) + L_0(\beta)K_{-1}(\beta)] \quad (13)$$

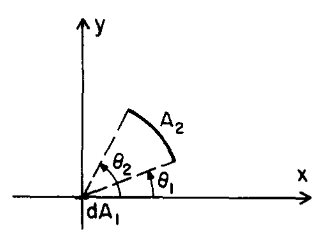

FIG. 2. Geometry and coordinate system for the 2-dim. fundamental solution.

with $\beta = 3/2a_\lambda R$ and $K_n(\beta)$ and $L_n(\beta)$ being the modified Bessel function and Sturve function respectively.

In a similar manner, the geometric mean beam lengths in the weak-band, strong-band, and very-strong-band limits for the geometry as shown in Fig. 2 can be evaluated. They are

$$R_{d1-2} = 4R/\pi \qquad (1.273\,R) \qquad (14)$$

$$S_{d1-2} = \left[\frac{4}{3\pi} F(\pi/2, 1/2^{1/2})\right]^2 2R \quad (1.238\,R) \quad (15)$$

$$W_{d1-2} = 2e^{-1/2} R \qquad (1.213\,R) \qquad (16)$$

where $F(\pi/2, 1/2^{1/2})$ is the elliptic integral of the first kind. It is important to note that for systems with 2-dim. geometry, it is impossible for the geometric total absorptance to stay entirely in the weak-band and strong-band limits. As some pathlengths become large, the geometric totally absorptance always reaches the very-strong-band limit. Mathematically, however, it can be readily shown that the contribution of elements with large pathlength to the surface integrals and line integrals in equation (2)–(4) is small. (In fact, this contribution becomes zero as the pathlength becomes infinite.) The error in assuming that the geometric total absorptance stays entirely in one limiting range over the whole surface is thus expected to be insignificant. Equations (14) and (15) can be interpreted physically as the weak-band and strong-band limits of the geometric mean beam length for a 2-dim. circular are in an 'average' sense. It is also interesting to note that equations (13)–(16) are independent of θ_1 and θ_2. They are thus applicable for any circular arc of radius R about the differential area dA. As demonstrated in reference [1], equations (14)–(16) can now be utilized together with the standard wide-band correlation to calculate the geometric mean absorptance of any 2-dim. object and the infinitesimal area dA.

EXAMPLE OF APPLICATION

As an example of application, the geometric mean beam length in the three limits for a circular cylinder of infinite length radiating to an element at its boundary is now evaluated. Utilizing the coordinate system as shown in Fig. 3, the relevant integrals are

$$R_{d1-2} = 2\int_0^{\pi/2} F_{d1-\phi} R_{d1-\phi} \qquad (17)$$

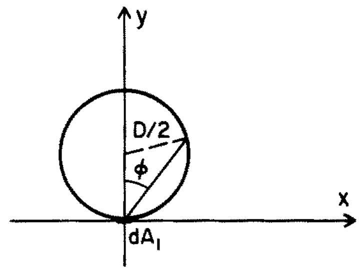

Fig. 3. Geometry of the example calculation.

$$S_{d1-2}^{1/2} = 2\int_0^{\pi/2} F_{d1-\phi} S_{d1-\phi}^{1/2} \qquad (18)$$

$$\ln W_{d1-2} = 2\int_0^{\pi/2} F_{d1-\phi} \ln W_{d1-\phi} \qquad (19)$$

where $dF_{d1-\phi} = \frac{1}{2}\sin\phi\, d\phi$ and $R_{d1-\phi}$, $S_{d1-\phi}$, and $W_{d1-\phi}$ are given by equations (14)–(16) with $R = D\cos\phi$. Equations (17)–(19) can be readily integrated to yield

$$R_{d1-2} = D \qquad (20)$$

$$S_{d1-2} = 0.946\,D \qquad (21)$$

$$W_{d1-2} = 0.893\,D. \qquad (22)$$

It is interesting to note that until now, the only reported value of the mean beam length values of this geometry is an empirical expression of 0.95 D [2] and the very-strong-band limit of 0.893 D [3].

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